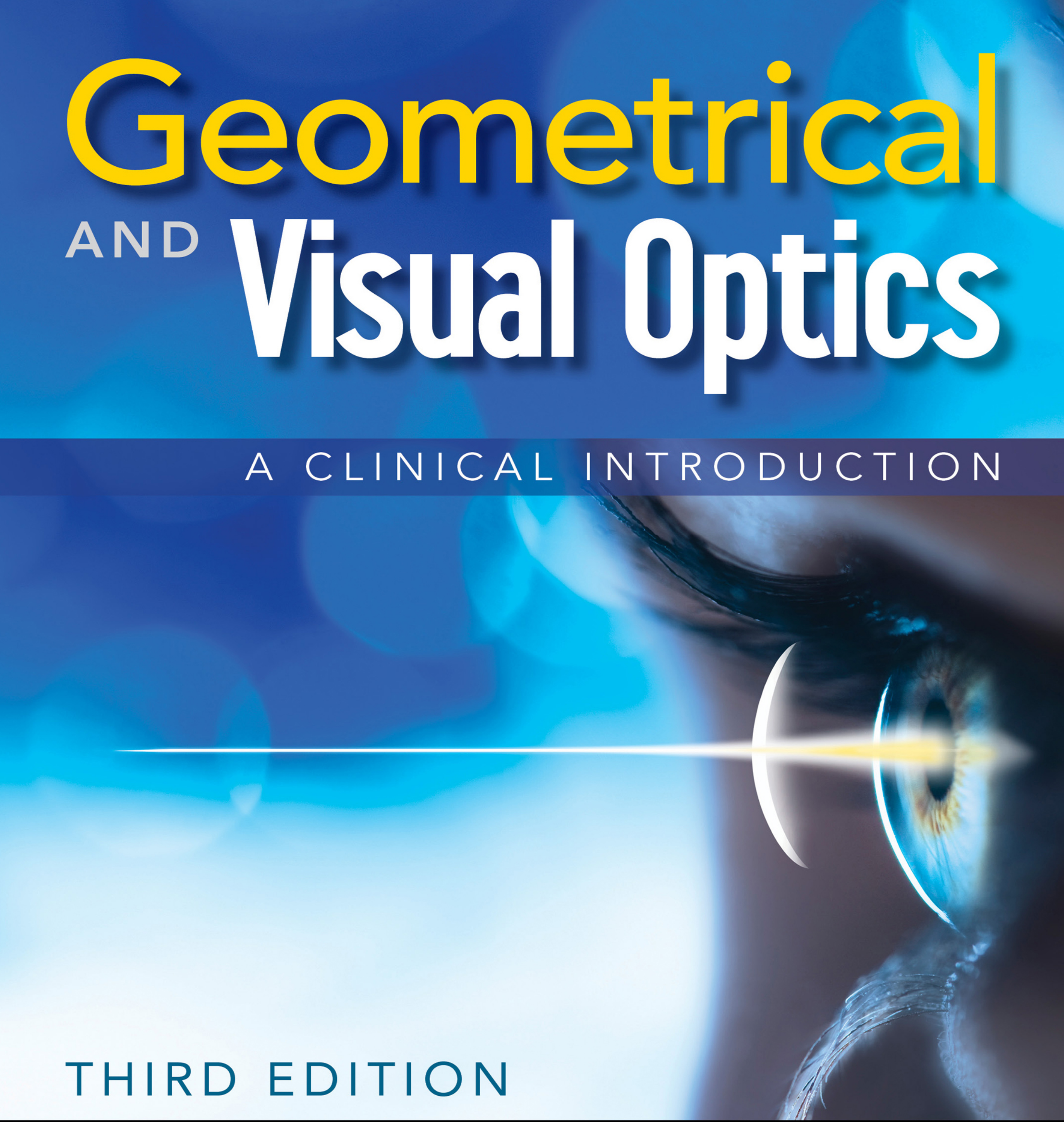

Geometrical AND Visual Optics

A CLINICAL INTRODUCTION

THIRD EDITION

McGraw Hill Education

STEVEN H. SCHWARTZ

Geometrical and Visual Optics

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A CLINICAL INTRODUCTION

THIRD EDITION

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Mc Graw Hill Education

New York Chicago San Francisco Athens London Madrid Mexico City
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Contents

Preface

…I strove that not one hour
Should idly pass. My eyes and mind took pride
In sacred Optics.[1]

Jan Vredeman De Vries

1527–c.1604

This book is intended as an approachable and appropriately rigorous introduction to geometrical and visual optics. It is meant to be a concise and learner-friendly resource for clinicians as they study optics for the first time and as they subsequently prepare for licensing examinations. The emphasis is on those optical concepts and problem-solving skills that underlie contemporary clinical eye care.

Because of its clinical utility, a vergence approach is stressed. While formulas are an inevitable part of optics, an attempt has been made to keep these to a minimum by emphasizing underlying concepts. Plentiful schematic figures and clinical examples are used to engage reader interest and foster understanding. Every effort is made to provide the reader with an intuitive and clinical sense of optics that will allow him or her to effectively care for patients.

The third edition has several new features designed to improve student learning. Figures have been upgraded and are in full color, and learning objectives are listed at the beginning of chapters. Clinical highlights are in color. A new chapter on lens thickness is included, and the chapter on prisms has been expanded to include vertical imbalance, bifocal jump and hand neutralization. Magnifying lenses and telescopes are now covered in two separate chapters. As in the previous edition, summaries, sample problems, and tables within chapters are color highlighted, and at the conclusion of each chapter, there is a brief summary and list of formulas. Throughout the book, sections have been rewritten and reorganized to make the material less intimidating and more comprehensible.

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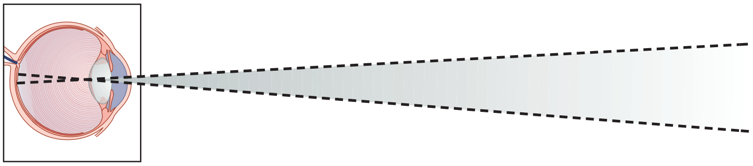
1. Vredeman De Vries, Jan (1604). *Studies in Perspective*. Republished in 2010 by Dover Publications, Inc.

To develop facility in geometrical and visual optics, it is necessary to solve problems. New end-of-chapter self-assessment problems have been incorporated in the third edition and detailed worked-out solutions to all problem elements are provided. To meet student demand for additional self-assessment tools, a third comprehensive practice examination (with answers) has been added to this edition. Between the 122 practice exam questions and approximately 218 end-of-chapter problem elements, there are now about 340 questions and problems of varying complexity. These are an integral part of the text.

It was my good fortune to be able to call upon knowledgeable and generous colleagues to review portions of earlier drafts of the second or third edition. The thoughtful input of Drs. Kathy Aquilante, Ian Bailey, Cliff Brooks, Jay Cohen, Geoffrey Goodfellow, Ralph Gundel, John Mark Jackson, Phil Kruger, Cristina Llerena Law, Nicole Putman, Jeff Rabin, Alan Reizman, Jie (Jason) Shen, and Frank Spors is greatly appreciated. Any shortcomings of the book are, of course, entirely my responsibility.



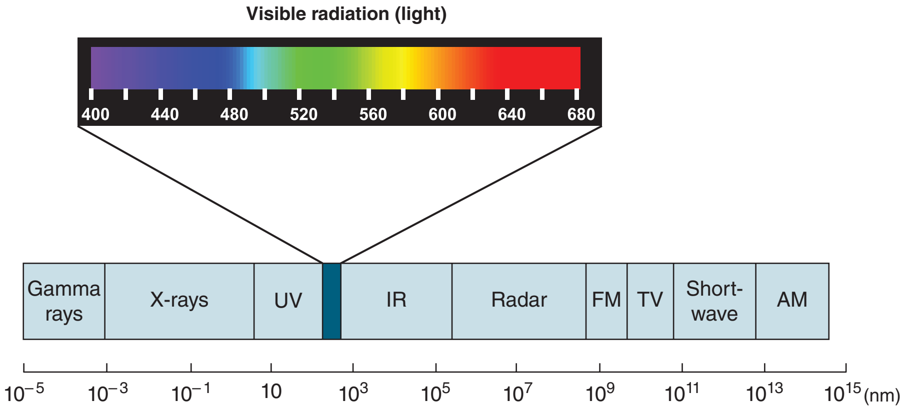
1

Basic Terms and Concepts

After completing this chapter you should be able to:

- Describe the EM spectrum and its relationship to light
- Describe the categories of UV radiation
- Recall the wavelengths that produce specific colors
- Describe the relationship between wavelength and energy, and explain its clinical implications
- Describe the relationship between light sources, rays and pencils, wavefronts, and vergence
- Determine object and image vergence
- Define and explain refractive index, and know those of commonly encountered materials
- Explain and apply Snell's law, including basic calculations
- Explain total internal reflection and its clinical implications, and calculate the critical angle

While we may think we're aware of what's going on around us, we're missing out on quite a bit. Our eyes are continuously bombarded by electromagnetic (EM) radiation, but as illustrated in Figure 1-1, we see only a small fraction of it. The remainder of the EM spectrum, including x-rays, ultraviolet (UV) and infrared radiation, and radar and radio waves, is invisible.



UV radiation
A (near): 320–400 nm
B: 280–320 nm
C: 100–280 nm

Visible radiation (light)
380–700 nm

Figure 1-1. Light (visible radiation), a small portion of the EM spectrum, ranges from about 380 to 700 nm. UV radiation, which because of its high energy contributes to the development of various ocular and skin conditions, can be classified as UVA, UVB, or UVC. (*Modified with permission from Schwartz SH*. Visual Perception: A Clinical Orientation. *5th ed. http://www.accessmedicine.com. Copyright © 2017 McGraw-Hill Education. All rights reserved. The colored visible radiation spectrum is used with permission from Dr. Jay Neitz.*)

EM radiation is specified by its wavelength. As can be seen in Figure 1-2, wavelength and frequency are inversely proportional—as the wavelength increases, frequency decreases (and vice versa).[1] They are related to each other as follows:

$$f = \frac{v}{\lambda}$$

where f is the frequency, v is the speed, and λ is the wavelength of the EM radiation.

Visible radiation—light—ranges from about 380 to 700 nm.[2] This radiation is absorbed by the retinal photopigments, setting in motion a complex chain of events that result in vision.[3]

---

[1] As light travels from a less dense material, such as air, to a more dense material, such as water, its frequency does not change, but its speed and wavelength decrease.

[2] One nanometer is equal to 10^{-9} m.

[3] For a basic introduction to visual processes see Schwartz SH. *Visual Perception: A Clinical Orientation*. 5th ed. New York: McGraw-Hill; 2017.

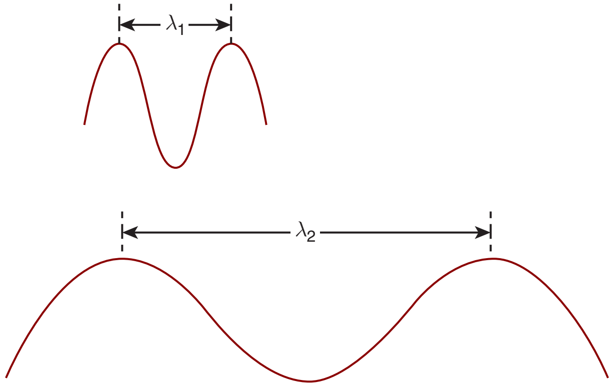


Figure 1-2. Wavelength (λ) and frequency are inversely proportional to each other. (*Adapted with permission from Schwartz SH.* Visual Perception: A Clinical Orientation. *5th ed. Copyright © 2017 McGraw-Hill Education. All rights reserved.*)

EM radiation is emitted in discrete packages of energy referred to as **photons** or **quanta.** The amount of energy in a photon is given by

$$E = hf$$

where E is the amount of energy per photon and h is Planck's constant.

By substitution, we have:

$$E = \frac{h\upsilon}{\lambda}$$

Clinical Highlight

As the wavelength decreases, the amount of energy per photon increases. For this reason, the absorption of short-wavelength radiation by body tissues is typically more damaging than the absorption of longer-wavelength radiation. The development of skin cancer, pinguecula, pterygium, photokeratitis, cataracts, and age-related macular degeneration has been linked to exposure to short-wavelength, high-energy UV radiation. Ocular exposure can be minimized by use of spectacles that block these rays and headgear (hats, visors) that protect the eye and its adnexa.

Longer-wavelength UV radiation may be categorized as either UVB, which ranges from 280 to 320 nm, or UVA, which ranges from 320 to 400 nm. UVB is absorbed by the skin epidermis resulting in sunburns. This radiation is most abundant during the summer months. In comparison, UVA, which penetrates deeper into the skin and is absorbed by the dermis, is present all year long. Accumulated damage to the dermis results in wrinkling of the skin and is responsible for commuter aging—wrinkling in areas that are exposed to sunlight (e.g., neck and back of hands) while driving to work. Both UVB and UVA have been associated with skin cancer.

SOURCES, LIGHT RAYS, AND PENCILS

For the study of geometrical and visual optics, we are interested primarily in the wave nature of light rather than its quantal nature. Figure 1-3 shows that a **point source**[4] of light, such as a star, emits concentric waves of light in much the same way a pebble dropped into a quiet pond of water generates waves of water. The peaks of the waves are called wavefronts. Think of them as circles with radii equal to the distance from the point source.

Let's look at this in more detail. Figure 1-4 shows wavefronts traveling from left to right. Consider these to be arcs of a circle whose center is the point source. As you can see, the curvature of these wavefronts decreases as the distance from the source increases. An arc with a longer radius is flatter than one with a shorter radius. At infinity (where the radius of the arc is infinity), the wavefronts are flat.

Note that direction of movement of the wavefronts in Figure 1-3 is represented by arrows—commonly called **light rays**—that are perpendicular to the wavefronts. A bundle of rays is called a **pencil**. As illustrated in Figure 1-5, the light rays that form a pencil can be diverging, converging, or parallel.

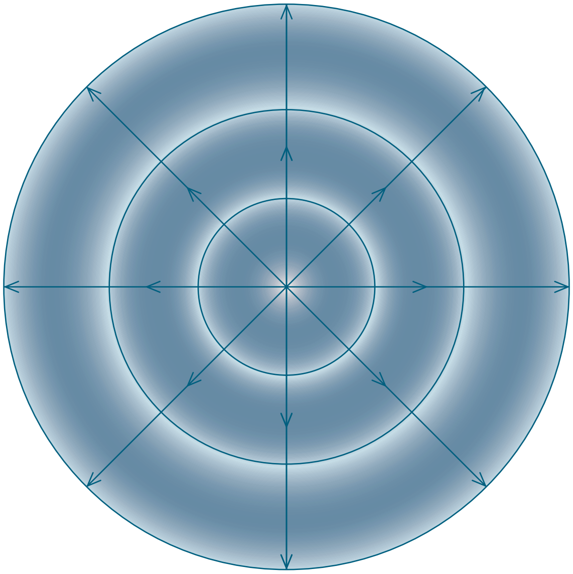


Figure 1-3. A point source of light emits concentric waves of light in much the same way a pebble dropped into a quiet pond of water produces waves of water. Light rays, represented by arrows, are perpendicular to the wavefronts.

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[4] The size of a point source approaches zero—it is infinitely small.

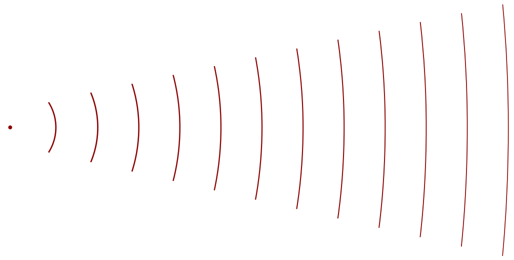

Figure 1-4. The curvature of wavefronts becomes less as the distance from the point source increases. They are arcs of a circle whose center is the point source. At infinity, the wavefronts are flat.

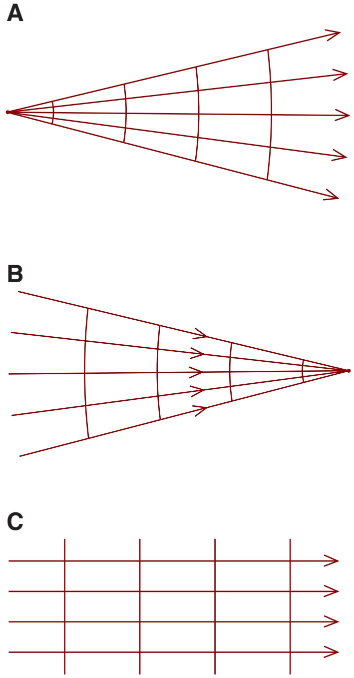

Figure 1-5. **A.** A diverging pencil of light rays emerges from a point source. **B.** A converging pencil of light rays is focused at a point. **C.** An object located at infinity produces a parallel pencil of light rays. Note that the light rays are perpendicular to the wavefronts.

A **diverging pencil** is produced by a point source of light, such as a star. When light rays are focused at a point, they create a **converging pencil.** A converging optical system (e.g., a magnifying lens) is required to create converging light. An object located infinitely far away forms a **parallel pencil** because, as we've seen in Figure 1-4, the wavefronts are flat (which means that the rays perpendicular to these wavefronts must be parallel to each other).

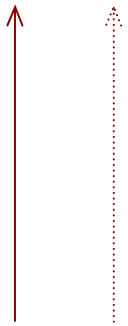

Figure 1-6. An extended object, such as an arrow, may be considered to consist of an infinite number of point sources. Each point emits diverging light rays.

An **extended source**, such as the arrow in Figure 1-6, is composed of an infinite number of point sources. Diverging light rays emerge from each of the point sources.

VERGENCE

When it comes to understanding and solving clinical optical problems, the concept of vergence goes a long way. At this point, I'll provide some working definitions that will get you going. Once we start looking at optical problems in subsequent chapters, vergence will become second nature to you (I hope!).

Vergence is a way to quantify the curvature of a wavefront. For point sources, curvature is greatest near the source and diminishes with distance from the source. The more curved a wavefront is, the greater its vergence. Likewise, the less curved it is, the less its vergence.

When solving optical problems, the vergence of diverging light is always—yes, *always*—labeled with a negative sign. The amount of divergence is quantified by taking the reciprocal of the distance *to* a point source. To arrive at the correct units for vergence—diopters (D)—the distance must be in meters. This may sound more difficult than it is. Figure 1-7, which gives vergence at three distances from a point source, should help. At 10.00 cm the vergence is −10.00 D, at 20.00 cm it is −5.00 D, and at 50.00 cm it is −2.00 D. In each case, we convert the distance to meters, take the reciprocal, and then label the vergence as negative to indicate that the light is diverging.[5] **Note the magnitude of the vergence (ignoring the sign) is greatest close to the source and diminishes as the distance increases.**

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[5] In Chapter 3, we'll learn that when light rays are in a substance other than air, the vergence is increased.

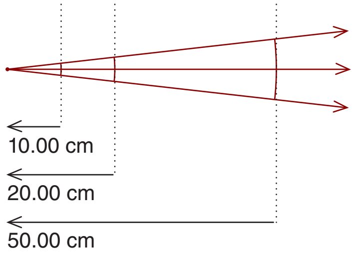

Figure 1-7. Diverging light rays have negative vergence. At distances of 10.00, 20.00, and 50.00 cm, the vergence is −10.00, −5.00, and −2.00 D, respectively. The magnitude of the vergence (ignoring the sign) *decreases* as the distance to the source *increases*.

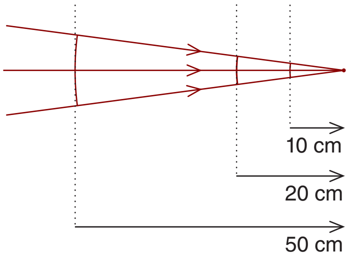

Figure 1-8. Converging light rays have positive vergence. At the distances of 10.00, 20.00, and 50.00 cm, the vergence is +10.00, +5.00, and +2.00 D, respectively. As the distance to the point of focus *increases*, convergence *decreases*.

As we mentioned previously, not all light is diverging. An optical system, such as a magnifying lens, can produce converging light. **To solve optical problems, the vergence of converging light is always—yes, *always*—labeled with a plus sign. It is quantified by taking the reciprocal of the distance (in meters) *to* the point where the light is focused.** Consider Figure 1-8, which shows light converging to a point focus. The vergence measured at distances of 10.00, 20.00, and 50.00 cm from this focus point is +10.00, +5.00, and +2.00 D, respectively. **Note the vergence is greatest close to the focus point and decreases as the distance increases.**

What is the vergence of a light source located infinitely far away? The wavefronts are flat—they have no curvature—making the vergence equal to zero. Thinking of it in quantitative terms, the reciprocal of the distance to the source (infinity) is zero. Or think of it this way: since the light rays are neither diverging nor converging, the vergence is zero. For clinical purposes, we normally consider distances equal to or greater than 20 ft (or 6 m) as infinitely far away.

REFRACTION AND SNELL'S LAW

The velocity of light depends on the medium in which it is traveling. Light travels more slowly in an optically dense medium, such as glass, than it does in a less dense medium, such as air. The degree to which an optical medium slows the velocity of light is given by its refractive index, which is the ratio of the speed of light in a vacuum to its speed in the medium. Refractive indices of materials commonly encountered in clinical practice are given in Table 1-1.

The change in velocity that occurs as light travels from one optical medium into another may cause a light ray to deviate from its original direction, a phenomenon referred to as **refraction**. Figure 1-9A illustrates the refraction that occurs when light traveling in air strikes a glass surface at an angle, θ, as measured with respect to the normal to the surface. The decrease in velocity causes the ray to change its direction. In this case, the light ray is refracted so that the angle made with the normal to the surface is decreased to θ'.

This illustrates a general rule you should memorize—when a light ray traveling in a material with a low index of refraction (an optically rarefied medium) enters a material with a higher index of refraction (an optically denser medium), the light ray is refracted *toward* (i.e., bent toward) the normal to the surface.

What occurs when light traveling in an optically dense medium enters one that is less dense? As can be seen in Figure 1-9B, the increase in velocity causes the light ray to be deviated away from the normal. Again, this is a handy fact to memorize.

It can be useful to quantify the refraction that occurs as light travels from one medium, which we'll call the **primary medium**, into another medium, which is called the **secondary medium**. Snell's law, which is given below, allows us to do so:

$$n(\sin \theta) = n'(\sin \theta')$$

where n is the index of refraction of the primary medium, n' is the index of refraction of the secondary medium, θ is the angle of incidence (with respect to the normal), and θ' is the angle of refraction (with respect to the normal).

TABLE 1-1. **REFRACTIVE INDICES OF COMMON MATERIALS**

Material	Refractive Index
Air	1.000
Water	1.333
Ophthalmic plastic (CR39)	1.498
Crown glass	1.523
Trivex	1.532
Polycarbonate	1.586
Essilor Airwear (plastic)	1.59
Essilor Thin & Lite (plastic)	1.67 or 1.74

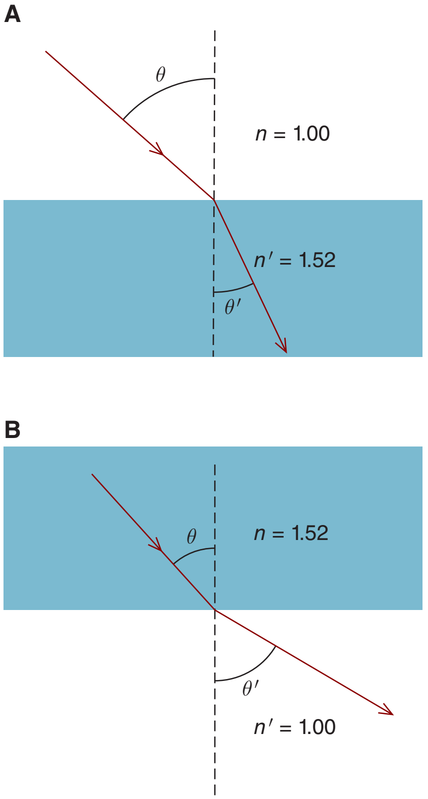

Figure 1-9. **A.** A light ray entering a denser medium is refracted *toward* the normal. **B.** A ray entering a rarer medium is refracted *away* from the normal.

Let's do a problem. *For a light ray traveling from air to crown glass, the angle of incidence is 20.00 degrees. What is the angle of refraction?*

In this and almost all optical problems, it's a good idea to draw a diagram. Figure 1-10 shows a light ray striking the glass surface such that it makes an angle of 20.00 degrees with the normal to the surface. Before doing the calculation, we know that the light ray is refracted toward the normal. How do we know this? As we mentioned earlier, when a light ray travels into a material with a higher index of refraction, it is deviated toward the normal. Snell's law allows us to determine the angle of refraction as follows:

$$n(\sin\theta) = n'(\sin\theta')$$

$$(1.00)(\sin 20.00^\circ) = (1.52)(\sin\theta')$$

$$\theta' = 13.00^\circ$$